Аудиторне вежбе из Рачунарског управљања

$z-$ трансформација

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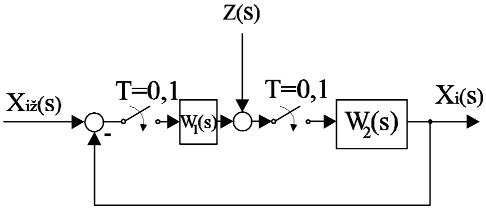
1. За систем који је задатат својим блок дијаграмом приказаним на слици одредити:



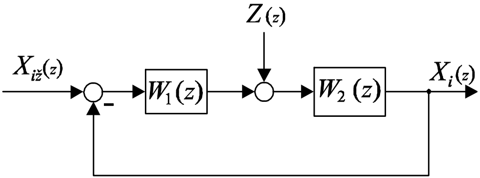
$W_1(s) = \frac{2}{s+1}$; $W_2(s) = \frac{s}{s+2}$; $X_{i\check{z}}(s) = \frac{1}{s}$; $Z(s) = \frac{1}{s}$

- Стабилност система помоћу модификованог Најквистовог критеријума.

Решење:

$Z-$ блок дијаграм посматраног система је:

Модификовани Најквистов критеријум



$W_1(s) = \frac{2}{s+1} \Longrightarrow W_1(z) = \mathcal{Z}\left\{\frac{2}{s+1}\right\} = 2\frac{z}{z-e^{-T}}$

$W_2(s) = \frac{s}{s+2} \Longrightarrow W_2(z) = \mathcal{Z}\left\{\frac{s}{s+2}\right\} = \mathcal{Z}\left\{1 - \frac{2}{s+2}\right\} = 1 - 2\frac{z}{z-e^{-2T}} = \frac{z-e^{-2T}-2z}{z-e^{-2T}} = \frac{-z-e^{-2T}}{z-e^{-2T}} \Longrightarrow W_{ok}(z) = W_1(z)\,W_2(z) = \frac{2z}{z-e^{-T}}\frac{-z-e^{-2T}}{z-e^{-2T}}$

Полови од $W_{ok}(z)$: $z - e^{-T} = 0 \Longrightarrow z_1^* = e^{-T} = e^{-0.1} = 0.905; z - e^{-2T} = 0 \Longrightarrow z_2^* = e^{-2T} = e^{-0.2} = 0,819 \Longrightarrow P = 0$

$W_{ok}(z) = \frac{-2z\left(z+e^{-0,2}\right)}{(z-e^{-0,1})(z-e^{-0,2})}$

Модификовани Најквистов критеријум

$$z = +\infty e^{j0} = +\infty \Longrightarrow W_{ok}\left(+\infty\right) = \frac{-2\infty\left(\infty + e^{-0,2}\right)}{(\infty - e^{-0,1})(\infty - e^{-0,2})} =$$

$$\frac{-2\infty\infty\left(1+\frac{e^{-0,2}}{\infty}\right)}{\infty\infty\left(1-\frac{e^{-0,1}}{\infty}\right)\left(1-\frac{e^{-0,2}}{\infty}\right)} = -2;$$

$$z = 1 + \rho e^{j\theta} \Longrightarrow W_{ok}\left(1+\rho e^{j\theta}\right) = \frac{-2\left(1+\rho e^{j\theta}\right)\left(1+\rho e^{j\theta}+e^{-0,2}\right)}{\left(1+\rho e^{j\theta}-e^{-0,1}\right)\left(1+\rho e^{j\theta}-e^{-0,2}\right)} =$$

$$\frac{-2\left(1+\rho e^{j\theta}\right)\left[\left(1+e^{-0,2}\right)+\rho e^{j\theta}\right]}{\left[(1-e^{-0,1})+\rho e^{j\theta}\right]\left[(1-e^{-0,2})+\rho e^{j\theta}\right]} =$$

$$\frac{-2\left[\left(1+e^{-0,2}\right)+\left(2+e^{-0,2}\right)\rho e^{j\theta}+\rho^2 e^{j2\theta}\right]}{\left[(1-e^{-0,1})(1-e^{-0,2})+(2-e^{-0,2}-e^{-0,1})\rho e^{j\theta}+\rho^2 e^{j2\theta}\right]} \approx$$

$$\frac{-3.6375-5.6375\rho e^{j\theta}}{\left[0.01725+0.27643\rho e^{j\theta}\right]} \Longrightarrow \frac{-3.6375-5.6375\cdot 0^{+}}{[0.01725+0.27643\cdot 0^{+}]} = \frac{-3.6375^{-}}{0.01725^{+}}$$

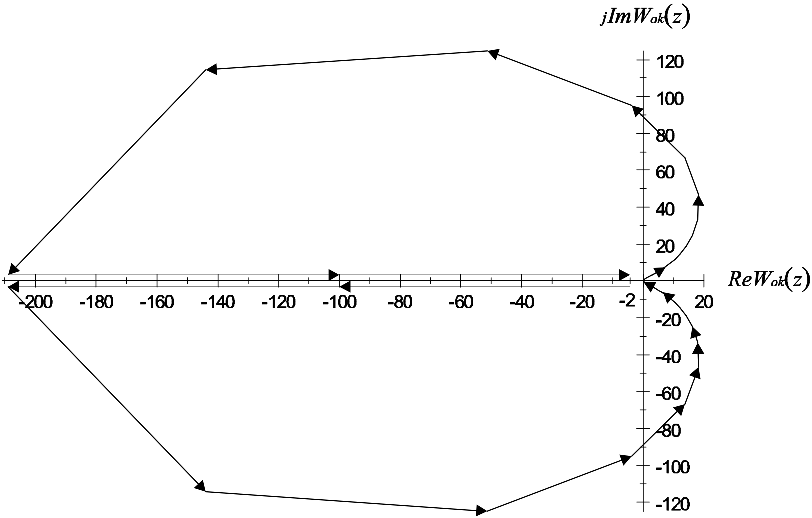
Модификовани Најквистов критеријум

z	$ReW_{ok}(z)$	$\mathrm{Im}\,W_{ok}(z)$	z	$ReW_{ok}(z)$	$\mathrm{Im}\,W_{ok}(z)$
$+\infty e^{j0}$	-2	0	$1\cdot e^{-j\frac{11\pi}{64}}$	8.9710	-9.6501
1^+	-210.87	0	$1\cdot e^{-j\frac{3\pi}{16}}$	7.7472	-8.0639
$1\cdot e^{-j\frac{\pi}{64}}$	-143.94	-114.33	$1\cdot e^{-j\frac{13\pi}{64}}$	6.7196	-6.8511
$1\cdot e^{-j\frac{\pi}{32}}$	-51.312	-124.89	$1\cdot e^{-j\frac{7\pi}{32}}$	5.8549	-5.9050
$1\cdot e^{-j\frac{3\pi}{64}}$	-3.7307	-95.234	$1\cdot e^{-j\frac{15\pi}{64}}$	5.1242	-5.1533
$1\cdot e^{-j\frac{\pi}{16}}$	13.683	-66.794	$1\cdot e^{-j\frac{\pi}{4}}$	4.5034	-4.5462
$1\cdot e^{-j\frac{5\pi}{64}}$	18.19	-46.721	$1\cdot e^{-j\frac{17\pi}{64}}$	3.9731	-4.0485
$1\cdot e^{-j\frac{3\pi}{32}}$	17.936	-33.444	$1\cdot e^{-j\frac{9\pi}{32}}$	3.5174	-3.6349
$1\cdot e^{-j\frac{7\pi}{64}}$	16.165	-24.674	$1\cdot e^{-j\frac{19\pi}{64}}$	3.1237	-3.2869
$1\cdot e^{-j\frac{\pi}{8}}$	14.09	-18.765	$1\cdot e^{-j\frac{5\pi}{16}}$	2.7816	-2.9909
$1\cdot e^{-j\frac{9\pi}{64}}$	12.136	-14.677	$1\cdot e^{-j\frac{21\pi}{64}}$	2.4829	-2.7364
$1\cdot e^{-j\frac{5\pi}{32}}$	10.425	-11.771	$1\cdot e^{-j\frac{11\pi}{32}}$	0.75840	-1.2744

Модификовани Најквистов критеријум

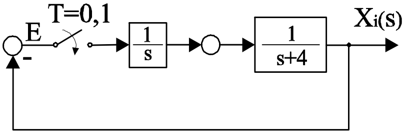
z	$ReW_{ok}(z)$	$\operatorname{Im} W_{ok}(z)$	z	$ReW_{ok}(z)$	$\operatorname{Im} W_{ok}(z)$
$1 \cdot e^{-j\frac{23\pi}{64}}$	-0.10465	0	$1 \cdot e^{-j\frac{35\pi}{64}}$	8.9710	9.6501
$1 \cdot e^{-j\frac{3\pi}{8}}$	0.75840	1.2744	$1 \cdot e^{-j\frac{9\pi}{16}}$	10.425	11.771
$1 \cdot e^{-j\frac{25\pi}{64}}$	2.4829	2.7364	$1 \cdot e^{-j\frac{37\pi}{64}}$	12.136	14.677
$1 \cdot e^{-j\frac{13\pi}{32}}$	2.7816	2.9909	$1 \cdot e^{-j\frac{19\pi}{16}}$	14.09	18.765
$1 \cdot e^{-j\frac{27\pi}{64}}$	3.1237	3.2869	$1 \cdot e^{-j\frac{39\pi}{64}}$	16.165	24.674
$1 \cdot e^{-j\frac{7\pi}{16}}$	3.5174	3.6349	$1 \cdot e^{-j\frac{5\pi}{8}}$	17.936	33.444
$1 \cdot e^{-j\frac{29\pi}{64}}$	3.9731	4.0485	$1 \cdot e^{-j\frac{41\pi}{64}}$	18.19	46.721
$1 \cdot e^{-j\frac{15\pi}{32}}$	4.5034	4.5462	$1 \cdot e^{-j\frac{21\pi}{16}}$	13.683	66.794
$1 \cdot e^{-j\frac{31\pi}{64}}$	5.1242	5.1533	$1 \cdot e^{-j\frac{125\pi}{64}}$	-3.7307	95.234
$1 \cdot e^{-j\frac{\pi}{2}}$	5.8549	5.9050	$1 \cdot e^{-j\frac{63\pi}{32}}$	-51.312	124.89
$1 \cdot e^{-j\frac{33\pi}{64}}$	6.7196	6.8511	$1 \cdot e^{-j\frac{127\pi}{64}}$	-143.94	114.33
$1 \cdot e^{-j\frac{17\pi}{32}}$	7.7472	8.0639	$1 \cdot e^{-j2\pi}$	-210.87	0

Модификовани Најквистов критеријум



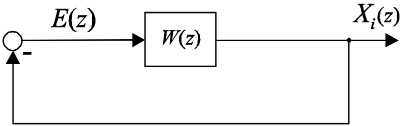
Модификовани Најквистов критеријум

1. Одредити грешку регулисане величине у САР чији је $s-$блок дијаграм приказан на слици:



Решење:

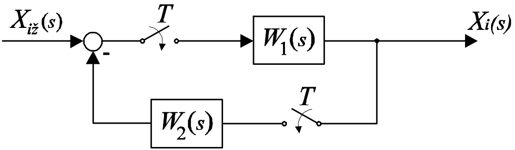
$Z-$ блок дијаграм посматраног система је:



$$W(z) = \overline{\mathcal{Z}}\left\{\frac{1}{s(s+4)}\right\} = \overline{\mathcal{Z}}\left\{\frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{1}{(s+4)}\right\} = \frac{1}{4}\overline{\mathcal{Z}}\left\{\frac{1}{s}\right\} - \frac{1}{4}\overline{\mathcal{Z}}\left\{\frac{1}{(s+4)}\right\} =$$
$$\frac{1}{4}\frac{z}{z-1} - \frac{1}{4}\frac{z}{z-e^{-0.4}}$$

Модификовани Најквистов критеријум

Одредити дискретну статичку грешку и појачање САУ са слике:

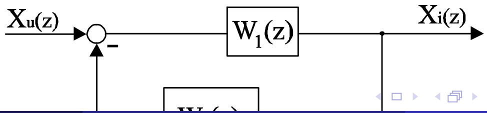


Слика:

при

$$W_1(s) = \frac{1}{s},\ W_2(s) = \frac{1}{s+2},\ T = 0,1\,\text{sec},\ X_{i\check{z}}(s) = \frac{1}{s}.$$

Решење:



Модификовани Најквистов критеријум

$W_1(s) = \frac{1}{s} \Longrightarrow W_1(z) = \overline{\mathcal{Z}}\left\{\frac{1}{s}\right\} = \frac{z}{z-1}$

$W_2(s) = \frac{1}{s+2} \Longrightarrow W_2(z) = \overline{\mathcal{Z}}\left\{\frac{1}{s+2}\right\} = \frac{z}{z-e^{-2T}} = \frac{z}{z-e^{-0.2}} \Longrightarrow$

$W(z) = \frac{W_1(z)}{1+W_1(z)W_2(z)} = \frac{\frac{z}{z-1}}{1+\frac{z}{z-1}\frac{z}{z-e^{-0.2}}} = \frac{z\left(z-e^{-0.2}\right)}{(z-1)(z-e^{-0.2})+z^2} =$
$\frac{0.5\left(z^2-e^{-0.2}z\right)}{z^2-\frac{1+e^{-0.2}}{2}z+\frac{e^{-0.2}}{2}} = \frac{0.5\left(z^2-e^{-0.2}z\right)}{z^2-0.90937z+0.40937} \Longrightarrow$

$z^*_{1,2} = 0.454\,69 \pm 0.450\,15i \Longrightarrow$систем је стабилан, па има смисла одређивати статичку грешку и појачање

Статичка грешка:
$\varepsilon^*_s = \lim_{k\longrightarrow\infty}\varepsilon(k) = \lim_{z\longrightarrow 1}(z-1)E(z)$

$E(z) = X_{i\check{z}}(z) - W_2(z)X_i(z), X_i(z) = W_1(z)E(z) \Longrightarrow E(z) =$
$X_{i\check{z}}(z) - W_1(z)W_2(z)E(z) \Longrightarrow E(z) = \frac{1}{1+W_1(z)W_2(z)}X_{i\check{z}}(z) =$
$\frac{1}{1+W_1(z)W_2(z)}\frac{z}{z-1} \Longrightarrow$

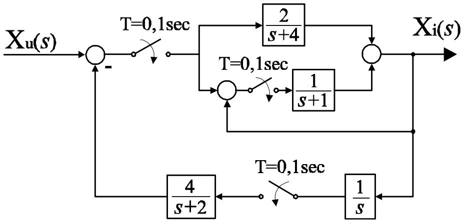
Модификовани Најквистов критеријум

$\varepsilon_s^* = \lim_{k\longrightarrow\infty} \varepsilon(k) = \lim_{z\longrightarrow 1} (z-1) E(z) =$
$\lim_{z\longrightarrow 1} (z-1) \frac{1}{1+W_1(z)W_2(z)} \frac{z}{z-1} = \lim_{z\longrightarrow 1} \frac{z}{1+W_1(z)W_2(z)} =$
$\lim_{z\longrightarrow 1} \frac{z}{1+\frac{z}{z-1}\frac{z}{z-e^{-0.2}}} = 0$

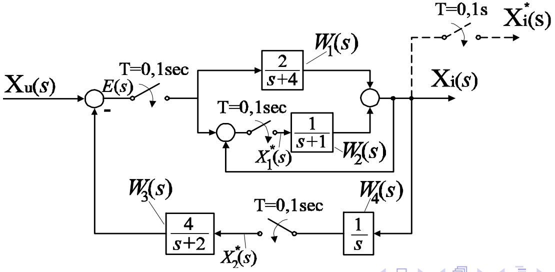
Појачање:

$K = \lim_{k\longrightarrow\infty} g(k) = \lim_{z\longrightarrow 1} (z-1) G(z), G(z) =$
$W(z) X_{i\check{z}}(z)\big|_{X_{i\check{z}}(z)=\frac{z}{z-1}} = \frac{W_1(z)}{1+W_1(z)W_2(z)} X_{i\check{z}}(z)\big|_{X_{i\check{z}}(z)=\frac{z}{z-1}} =$
$\frac{\frac{z}{z-1}}{1+\frac{z}{z-1}\frac{z}{z-e^{-0.2}}} \frac{z}{z-1} = \frac{z\left(z-e^{-0.2}\right)}{(z-1)(z-e^{-0.2})+z^2} \frac{z}{z-1} \Longrightarrow K =$
$\lim_{z\longrightarrow 1} \frac{z^2\left(z-e^{-0.2}\right)}{(z-1)(z-e^{-0.2})+z^2} = 1 - e^{-0.2} = 0.181\,27$

1. Одредити појачање система са слике.



Решење:



Модификовани Најквистов критеријум

$X_i(s) = W_1(s)E^*(s) + W_2(s)X_1^* \Longrightarrow X_i^*(s) = W_1^*(s)E^*(s) + W_2^*(s)X_1^*$

$X_1(s) = E^*(s) + X_i(s) \Longrightarrow X_1^*(s) = E^*(s) + X_i^*(s)$

$E(s) = X_u(s) - W_3(s)X_2^*(s) \Longrightarrow E^*(s) = X_u^*(s) - W_3^*(s)X_2^*(s)$

$X_2(s) = W_4(s)X_i(s) = W_1(s)W_4(s)E^*(s) + W_2(s)W_4(s)X_1^* \Longrightarrow$
$X_2^*(s) = W_1W_4^*(s)E^*(s) + W_2W_4^*(s)X_1^*$

$\Longrightarrow$
$\left\{1 + \frac{[W_1^*(s)+W_2^*(s)]W_3^*(s)W_2W_4^*(s)}{1+W_3^*(s)[W_1W_4^*(s)+W_2W_4^*(s)]} - W_2^*(s)\right\}X_i^*(s) =$
$\frac{W_1^*(s)+W_2^*(s)}{1+W_3^*(s)[W_1W_4^*(s)+W_2W_4^*(s)]}X_u^*(s) \Longrightarrow X_i^*(s) =$
$\frac{W_1^*(s)+W_2^*(s)}{[1-W_2^*(s)]\{1+W_3^*(s)[W_1W_4^*(s)+W_2W_4^*(s)]\}+[W_1^*(s)+W_2^*(s)]W_3^*(s)W_2W_4^*(s)}X_u^*(s) \Longrightarrow$
$X_i(z) =$
$\frac{W_1(z)+W_2(z)}{[1-W_2(z)]\{1+W_3(z)[W_1W_4(z)+W_2W_4(z)]\}+[W_1(z)+W_2(z)]W_3(z)W_2W_4(z)}X_u(z);$

$W_1(z) = \mathcal{Z}\left\{\frac{2}{s+4}\right\} = \frac{2z}{z-e^{-0,4}}$; $W_2(z) = \mathcal{Z}\left\{\frac{1}{s+1}\right\} = \frac{z}{z-e^{-0,1}}$; $W_3(z) =$
$\mathcal{Z}\left\{\frac{4}{s+2}\right\} = \frac{4z}{z-e^{-0,2}}$; $W_4(z) = \mathcal{Z}\left\{\frac{1}{s}\right\} = \frac{z}{z-1}$; $W_1W_4(z) = \mathcal{Z}\left\{\frac{2}{s(s+4)}\right\} =$

Модификовани Најквистов критеријум

$\overline{\mathcal{Z}}\left\{\frac{0,5}{s}-\frac{0,5}{s+4}\right\}=\overline{\mathcal{Z}}\left\{\frac{0,5}{s}\right\}-\overline{\mathcal{Z}}\left\{\frac{0,5}{s+4}\right\}=\frac{0,5\left(1-e^{-0,4}\right)z}{(z-1)(z-e^{-0,4})}$

$W_2W_4(z)=\overline{\mathcal{Z}}\left\{\frac{1}{s(s+1)}\right\}=\overline{\mathcal{Z}}\left\{\frac{1}{s}-\frac{1}{s+1}\right\}=\overline{\mathcal{Z}}\left\{\frac{1}{s}\right\}-\overline{\mathcal{Z}}\left\{\frac{1}{s+1}\right\}=$
$\frac{\left(1-e^{-0,1}\right)z}{(z-1)(z-e^{-0,1})}$

$W(z)=\frac{W_1(z)+W_2(z)}{[1-W_2(z)]\{1+W_3(z)[W_1W_4(z)+W_2W_4(z)]\}+[W_1(z)+W_2(z)]W_3(z)W_2W_4(z)}=$
$\frac{z\left(3z-2e^{-0,1}-e^{-0,4}\right)(z-1)\left(z-e^{-0,1}\right)\left(z-e^{-0,2}\right)}{q(z)}$

$q(z)=$
$-e^{-0,1}\left[z^2-\left(1+e^{-0,1}\right)z+e^{-0,1}\right]\left[z^2-\left(e^{-0,2}+e^{-0,4}\right)z+e^{-0,6}\right]+$
$4\left(-1,5e^{-0,1}+0,5e^{-0,5}+e^{-0,2}\right)z^3-$
$4\left(1,5e^{-0,6}-e^{-0,5}-0,5e^{-0,2}\right)z^2+$
$12\left(1-e^{-0,1}\right)z^4-4\left(2e^{-0,1}+e^{-0,4}-2e^{-2}-e^{-5}\right)z^3$

$W(z)=\frac{0.466\,86z\left(3z-2e^{-0,1}-e^{-0,4}\right)(z-1)\left(z-e^{-0,1}\right)\left(z-e^{-0,2}\right)}{z^4-2.464\,5z^3+2.362\,7z^2-1.117\,1z+0.231\,84}\Longrightarrow z_1^*=$
$0.904\,33,\ z_2^*=0.744\,05,\ z_3^*=0.408\,06-0.421\,95i,$
$z_4^*=0.408\,06+0.421\,95i$

$\left|z_i^*\right|<1,\forall i=1,2,3,4\left(\left|z_{4,5}^*\right|=0.586\,99\right)\Longrightarrow$ Систем је стабилан

Модификовани Најквистов критеријум

$\Longrightarrow$Има смисла одређивање појачања!

$$K = \lim_{k \longrightarrow \infty} g(k) = \lim_{z \longrightarrow 1} (z-1)\, G(z),\ G(z) =$$

$$W(z)\, X_u(z)\big|_{X_u(z)=\frac{z}{z-1}} = \frac{0.466\,86 z^2 \left(3z - 2e^{-0,1} - e^{-0,4}\right)\left(z - e^{-0,1}\right)\left(z - e^{-0,2}\right)}{z^4 - 2.\,464\,5 z^3 + 2.\,362\,7 z^2 - 1.\,117\,1 z + 0.231\,84} \Longrightarrow$$

$$K = \lim_{z \longrightarrow 1} (z-1) \frac{0.466\,86 z^2 \left(3z - 2e^{-0,1} - e^{-0,4}\right)\left(z - e^{-0,1}\right)\left(z - e^{-0,2}\right)}{z^4 - 2.\,464\,5 z^3 + 2.\,362\,7 z^2 - 1.\,117\,1 z + 0.231\,84} = 0$$

2. Одредити ε_s^* за САУ описан са:

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ -0,25 & 0,7 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (y(k) + 2z(k)),$$
$$x_i = x_1,$$
$$q(k+1) = q(k) + 5\varepsilon(k), y(k) = 2q(k) + \varepsilon(k).$$

Решење:

$\mathcal{Z}\{x(k+1)\} = \mathcal{Z}\{Ax(k)\} + \mathcal{Z}\{b(y(k) + 2z(k))\} \Longrightarrow zX(z) = AX(z) + bY(z) + 2bZ(z) \Longrightarrow (zI - A)^{-1} / (zI - A) X(z) = bY(z) + 2bZ(z) \Longrightarrow X(z) = (zI - A)^{-1} bY(z) + 2(zI - A)^{-1} bZ(z)$
$x_i = x_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} x = c^T x \Longrightarrow \mathcal{Z}\{x_i\} = \mathcal{Z}\left\{c^T x\right\} \Longrightarrow X_i(z) = c^T X(z) = c^T (zI - A)^{-1} bY(z) + 2c^T (zI - A)^{-1} bZ(z) = W_{oY}(z) Y(z) + W_{oZ}(z) Z(z) \Longrightarrow W_{oY}(z) = c^T (zI - A)^{-1} b, W_{oZ}(z) = 2c^T (zI - A)^{-1} bZ(z) = 2W_{oY}(z)$
$(zI - A) = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -0,25 & 0,7 \end{pmatrix} =$

$$\begin{pmatrix} z & -1 \\ 0,25 & z-0,7 \end{pmatrix} \Longrightarrow \det \begin{pmatrix} z & -1 \\ 0,25 & z-0,7 \end{pmatrix} =$$

$$z^2 - 0.7z + 0.25, \mathrm{adj} \begin{pmatrix} z & -1 \\ 0,25 & z-0,7 \end{pmatrix} = \begin{pmatrix} z-0,7 & 1 \\ -0,25 & z \end{pmatrix} \Longrightarrow$$

$$(zI - A)^{-1} = \frac{\mathrm{adj}(zI-A)}{\det(zI-A)} = \begin{pmatrix} \frac{z-0,7}{z^2-0.7z+0.25} & \frac{1}{z^2-0.7z+0.25} \\ -\frac{0,25}{z^2-0.7z+0.25} & \frac{z}{z^2-0.7z+0.25} \end{pmatrix}$$

$$W_{oY}(z) = c^T (zI - A)^{-1} b =$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{z-0,7}{z^2-0.7z+0.25} & \frac{1}{z^2-0.7z+0.25} \\ -\frac{0,25}{z^2-0.7z+0.25} & \frac{z}{z^2-0.7z+0.25} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{z-0,7}{z^2-0.7z+0.25} & \frac{1}{z^2-0.7z+0.25} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{z^2-0.7z+0.25};$$

$$W_{oZ}(z) = \frac{2}{z^2-0.7z+0.25}$$

$$\mathcal{Z}\{q(k+1)\} = \mathcal{Z}\{q(k)\} + \mathcal{Z}\{5\varepsilon(k)\} \Longrightarrow zQ(z) = Q(z) + 5E(z) \Longrightarrow$$

$$(z-1)Q(z) = 5E(z) \Longrightarrow Q(z) = \frac{5}{z-1}E(z); \mathcal{Z}\{y(k)\} =$$

$$2\mathcal{Z}\{q(k)\} + \mathcal{Z}\{\varepsilon(k)\} \Longrightarrow Y(z) = 2Q(z) + E(z) =$$

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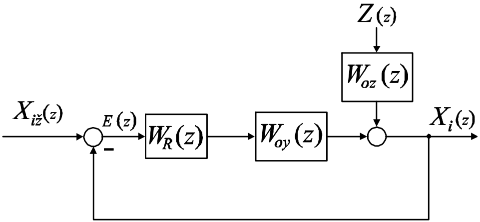
$2\frac{5}{z-1}E(z)+E(z)=W_R(z)E(z)=\frac{z+9}{z-1}E(z)$
$W_R(z)=\frac{z+9}{z-1}$



$W_{x_{iž}}(z)=\frac{W_R(z)W_{oy}(z)}{1+W_R(z)W_{oy}(z)}$; $W_z(z)=\frac{W_{oz}(z)}{1+W_R(z)W_{oy}(z)}\Longrightarrow$
$1+W_R(z)W_{oy}(z)=1+\frac{z+9}{z-1}\frac{1}{z^2-0.7z+0.25}=\frac{(z-1)\left(z^2-0.7z+0.25\right)+z+9}{(z-1)(z^2-0.7z+0.25)}$, $1+W_R(z)W_{oy}(z)=0\Longrightarrow(z-1)\left(z^2-0.7z+0.25\right)+z+9=0\Longrightarrow$
$z^3-1.7z^2+1.95z+8.75=0\Longrightarrow z_1^*=1.5478+1.9683i$, $z_2^*=1.5478-1.9683i$, $z_3^*=-1.3955$, Постоји пол по модулу већи од 1$\Longrightarrow$систем није стабилан$\Longrightarrow$ нема смисла одређивање статичке грешке.

Модификовани Најквистов критеријум

3. Испитати осмотривост система са слике из задатка бр.1.
   Решење:
   $W(z) = \frac{0.46686z\left(3z-2e^{-0,1}-e^{-0,4}\right)(z-1)\left(z-e^{-0,1}\right)\left(z-e^{-0,2}\right)}{z^4-2.4645z^3+2.3627z^2-1.1171z+0.23184} = \frac{X_i(z)}{X_u(z)} \Longrightarrow$
   $A = \begin{pmatrix} -a_3 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_1 & 0 & 0 & 1 \\ -a_0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2.4645 & 1 & 0 & 0 \\ -2.3627 & 0 & 1 & 0 \\ 1.1171 & 0 & 0 & 1 \\ -0.23184 & 0 & 0 & 0 \end{pmatrix}; c =$
   $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T; S = \begin{pmatrix} c^T & A^T c^T & \left(A^T\right)^2 c^T & \left(A^T\right)^3 c^T \end{pmatrix} =$
   $\begin{pmatrix} 1 & 2.4645 & 3.7111 & 4.4401 \\ 0 & 1 & 2.4645 & 3.7111 \\ 0 & 0 & 1 & 2.4645 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Longrightarrow$

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$$\text{rang}\, S = \text{rang}\begin{pmatrix} 1 & 2.4645 & 3.7111 & 4.4401 \\ 0 & 1 & 2.4645 & 3.7111 \\ 0 & 0 & 1 & 2.4645 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 4 \Longrightarrow$$ Систем je осмотрив.

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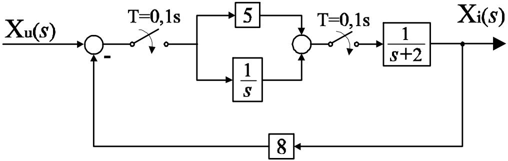
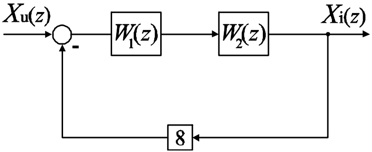
4. По дефиницији испитати привлачење $x_r = 0_x$ система са слике:

$\Longrightarrow$



Решење:



$W_1(z) = \overline{\mathcal{Z}}\left\{5+\frac{1}{s}\right\} = 5 + \frac{z}{z-1} = \frac{6z-5}{z-1}$; $W_2(z) = \overline{\mathcal{Z}}\left\{\frac{1}{s+2}\right\} = \frac{z}{z-e^{-0.2}} \Longrightarrow W(z) = \frac{W_1(z)W_2(z)}{1+8W_1(z)W_2(z)} = \frac{0.122\,45z^2-0.102\,04z}{z^2-0.853\,44z+1.670\,9\times10^{-2}} \Longrightarrow$

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$x_i(k+2) - 0.85344x(k+1) + 1.6709 \times 10^{-2}x_i(k) =$
$0.12245x_u(k+2) - 0.10204x_u(k+1)$

$A = \begin{pmatrix} -a_1 & 1 \\ -a_0 & 0 \end{pmatrix} = \begin{pmatrix} 0.85344 & 1 \\ -1.6709 \times 10^{-2} & 0 \end{pmatrix}; b = \begin{pmatrix} b_1 - a_1 b_2 \\ b_0 - a_0 b_2 \end{pmatrix} =$
$\begin{pmatrix} 2.4637 \times 10^{-3} \\ -2.046 \times 10^{-3} \end{pmatrix}; c = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T; d = b_2 = 0.12245 \Longrightarrow$

$x(k+1) = Ax(k) + bx_u(k) \Longrightarrow \mathcal{Z}\{x(k+1)\} =$
$\mathcal{Z}\{Ax(k)\} + b\mathcal{Z}\{x_u(k)\} \Longrightarrow zX(z) - \underset{=0}{x(0)} = AX(z) + bX_u(z)$

$(zI - A)^{-1} / (zI - A) X(z) = x(0) + bX_u(z) \Longrightarrow X(z) =$
$(zI - A)^{-1} x(0); zI - A = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 0.85344 & 1 \\ -1.6709 \times 10^{-2} & 0 \end{pmatrix} =$
$\begin{pmatrix} z - 0.85344 & -1 \\ 1.6709 \times 10^{-2} & z \end{pmatrix};$

$\det(zI - A) = \det \begin{pmatrix} z - 0.85344 & -1 \\ 1.6709 \times 10^{-2} & z \end{pmatrix} =$
$z^2 - 0.85344z + 1.6709 \times 10^{-2}; \operatorname{adj}(zI - A) =$

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$$\mathrm{adj}\begin{pmatrix} z-0.85344 & -1 \\ 1.6709\times10^{-2} & z \end{pmatrix} = \begin{pmatrix} z & 1 \\ -1.6709\times10^{-2} & z-0.85344 \end{pmatrix}$$

$$(zI-A)^{-1} = \frac{\mathrm{adj}(zI-A)}{\det(zI-A)} =$$

$$\begin{pmatrix} \frac{z}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{1}{(z-0.83339)(z-2.0049\times10^{-2})} \\ \frac{-1.6709\times10^{-2}}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{z-0.85344}{(z-0.83339)(z-2.0049\times10^{-2})} \end{pmatrix} \Longrightarrow X(z) =$$

$$\begin{pmatrix} \frac{z}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{1}{(z-0.83339)(z-2.0049\times10^{-2})} \\ \frac{-1.6709\times10^{-2}}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{z-0.85344}{(z-0.83339)(z-2.0049\times10^{-2})} \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} \Longrightarrow$$

$$\chi(k;x_0) =$$

$$\mathcal{Z}^{-1}\left\{\begin{pmatrix} \frac{z}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{1}{(z-0.83339)(z-2.0049\times10^{-2})} \\ \frac{-1.6709\times10^{-2}}{(z-0.83339)(z-2.0049\times10^{-2})} & \frac{z-0.85344}{(z-0.83339)(z-2.0049\times10^{-2})} \end{pmatrix}\right\}.$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} =$$

$$\begin{pmatrix} \mathcal{Z}^{-1}\left\{\frac{z}{(z-0.83339)(z-2.0049\times10^{-2})}\right\} & \mathcal{Z}^{-1}\left\{\frac{1}{(z-0.83339)(z-2.0049\times10^{-2})}\right\} \\ \mathcal{Z}^{-1}\left\{\frac{-1.6709\times10^{-2}}{(z-0.83339)(z-2.0049\times10^{-2})}\right\} & \mathcal{Z}^{-1}\left\{\frac{z-0.85344}{(z-0.83339)(z-2.0049\times10^{-2})}\right\} \end{pmatrix}.$$

Модификовани Најквистов критеријум

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} \chi_1(k;x_0) \\ \chi_1(k;x_0) \end{pmatrix}; \chi_1(k;x_0) =$$
$$\mathcal{Z}^{-1}\left\{\frac{z}{(z-0.833\,39)(z-2.004\,9\times10^{-2})}\right\}x_1(0) +$$
$$\mathcal{Z}^{-1}\left\{\frac{1}{(z-0.833\,39)(z-2.004\,9\times10^{-2})}\right\}x_2(0);$$
$$\chi_2(k;x_0) = \mathcal{Z}^{-1}\left\{\frac{-1.670\,9\times10^{-2}}{(z-0.833\,39)(z-2.004\,9\times10^{-2})}\right\}x_1(0) +$$
$$\mathcal{Z}^{-1}\left\{\frac{z-0.853\,44}{(z-0.833\,39)(z-2.004\,9\times10^{-2})}\right\}x_2(0) \Longrightarrow \|\chi(k;x_0)\| =$$
$$\sqrt{[\chi_1(k;x_0)]^2 + [\chi_2(k;x_0)]^2}$$
$$\lim_{k\to\infty}\|\chi(k;x_0)\| = \lim_{k\to\infty}\sqrt{[\chi_1(k;x_0)]^2 + [\chi_2(k;x_0)]^2} = ?$$